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THE THERMAL ANALYSIS OF A BELT TYPE RADIATOR BY THE METHOD OF MATCHED ASYMPTOTIC EXPANSIONS

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Abstract—Analytical expressions are obtained for the temperature distribution and heat transfer capacity of a belt type radiator. The problem is shown to reduce to two second-order ordinary differential equations whose solutions are coupled by two common sets of boundary conditions. One of these equations is non-linear and cannot be solved analytically in terms of known functions. Singular perturbation theory is employed to derive uniformly valid solutions of this equation for small values of the radiation–conduction parameter, ε. In order to exhibit the qualitative trends of these expressions, the temperature distribution, correct to second-order, is calculated for a representative system.

NOMENCLATURE

a, length of belt segment in contact with condenser drum;
A, constant, defined in equation (24);
B, constant, defined in equation (24);
C_n, constants;
C_p, specific heat of belt material;
h, thermal contact coefficient;
k, thermal conductivity of the belt material;
l, total belt length;
$\dot{Q}$, nondimensional heat transfer capacity of the radiator, $= (\dot{q}/hWl^{3/4})(\sigma\bar{\varepsilon}/\rho v c_p t)^{1/4}$;
$\dot{q}$, heat transfer capacity of the radiator;
T, belt temperature;
T_D, condenser drum temperature;
t, belt thickness;
v, belt velocity;
W, belt width;
X, nondimensional length coordinate;
x, length coordinate.

Greek symbols

α, parameter defined as the ratio of the energy flux from the drum to the belt over the convected energy flux, $= (hl/\rho v c_p t)$;
$\bar{\varepsilon}$, total hemispherical emittance of the outer belt surface;
ε, parameter defined as the ratio of conductive energy flux to radiative energy flux, $= (k/\rho v c_p l)$;
θ, nondimensional temperature;
θ_D, nondimensional condenser drum temperature;
θ_n, n-th order nondimensional temperature;
ρ, density of the belt material;
σ, Stefan–Boltzmann constant.

1. INTRODUCTION

THE OPERATION of spaceborne closed cycle powerplants requires that all of the degraded thermal energy not converted into work must be rejected to the environment. In space the only energy transfer mechanism available to accomplish this rejection is thermal radiation. Most conventional space powerplant designs employ the circulation of either the working fluid or a secondary heat exchange fluid to transport waste energy to one or more radiators. These radiators usually consist of supply and return manifolds which are connected by a network of tubing that forms the actual radiating surfaces. Because of the large amount of hardware required, radiators of this type impose severe weight penalties on the powerplant designer. They may, for example, comprise one-half of the overall powerplant weight for power levels above 1 MW [1]. There are also a number of reliability problems associated with liquid-filled radiators such as leakage due to component failures or meteoroid punctures and freezing of the working fluid during periods of minimal power output.

In an effort to circumvent the inherent problems of liquid-filled radiators, Weatherston and Smith [1] proposed a novel device called the belt, or "moving fin", radiator. Both Weatherston and Smith and, subsequently, Burge [2] have demonstrated that reductions in radiator weight of up to 60 per cent are attainable by means of this concept.

This radiator consists of two primary components: (1) a long flexible belt, and (2) a condenser drum which is heated by the waste energy of the powerplant. The coolest part of the belt is brought into contact with the drum where energy is transferred to the belt by conduction, thereby raising the temperature of the belt to its maximum value. As an element of the belt moves away from the drum its temperature is reduced, mainly by radiation to the environment, and to some extent by conduction along the belt. Thus, in steady-state operation an overall balance is achieved between the energy transferred from the drum to the belt by conduction and the energy transferred from the belt to the environment by radiation.

Two basic configurations of the belt radiator have been proposed. The original system of Weatherston and Smith [1] shown in Fig. 1a employs a revolving condenser drum. The so-called "revolving belt" system envisioned by Burge [2], however, uses a fixed condenser drum with the entire belt itself revolving about the drum as shown in Fig. 1b. Burge has shown that the revolving belt system is superior for powerplant outputs between one and ten megawatts, the revolving drum system best for outputs greater than ten megawatts, and that both systems perform equally well for outputs less than one megawatt. The analysis presented in this paper applies to both of these configurations.

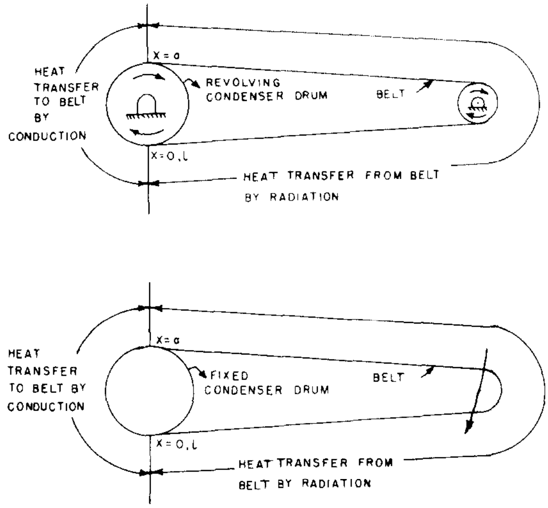

FIG. 1. Basic configurations of the belt radiator.
(a) Revolving drum
(b) Revolving belt

The analyses of Weatherston and Smith [1, 3] are based upon simplified calculations that are directed at exhibiting the weight-saving characteristics of the belt radiator system. McGean [4] performed a regular perturbation analysis appropriate to the "weak radiation" regime. It will be shown in this paper, however, that typical systems will operate in the "weak conduction" rather than the "weak radiation" regime. Burge [2] completed a more detailed analysis in which conduction in the belt is completely neglected without rationally assessing the effect of this assumption. As shown in the analysis, taking conduction into account changes the problem from a regular to a singular perturbation problem. It was, therefore, felt that this effect should be investigated since past experience has shown that singular perturbation problems sometimes yield rather unexpected results.

Other investigators have used singular perturbation theory to solve combined conduction–radiation problems of this type. Lick [5] examined the case of a radiating gas contained between two parallel plates, while Mueller and Malmuth [6] and Malmuth, Kascic and Mueller [7] investigated the temperature distributions in radiating heat shields.

2. ANALYSIS

2.1 *Formulation of the problem*

The analysis is based on the following assumptions:

(1) The heat transfer from the condenser drum to the moving belt may be characterized by a "thermal contact coefficient", h.
(2) The region of heat transfer to the drum is shielded such that no radiation losses occur in this zone.
(3) The belt velocity is a constant.
(4) The thermal properties of the belt are constant.
(5) Radiation from the belt takes place only on the outside surface of the belt.
(6) The belt is radiating to a sink at absolute zero temperature.
(7) The temperature gradient across the thickness of the belt and "end effects" at the sides are negligible; that is, the temperature distribution is one-dimensional.
(8) The condenser drum temperature is a constant.
(9) The local radius of curvature of the belt is everywhere large enough such that the belt may be adequately described by a rectangular coordinate system.

Performing an energy balance on a differential element of the belt that is in contact with the drum ($0 \leqslant x \leqslant a^-$), we obtain

$$h(T - T_D) - kt\frac{d^2T}{dx^2} + \rho v c_p t\frac{dT}{dx} = 0 \qquad (1)$$

where the first term is the energy flux from the drum to the belt, the second term is the conductive energy flux, and the third term is the convected energy flux. Similarly, for a differential element of the belt not in contact with the drum ($a^+ \leqslant x \leqslant l$), we have

$$\sigma\bar{\varepsilon}T^4 - kt\frac{d^2T}{dx^2} + \rho v c_p t\frac{dT}{dx} = 0 \qquad (2)$$

where the first term is the radiative energy flux from the belt to the surroundings and the second and third terms retain the same meanings as in equation (1). The symbols are defined in the Nomenclature. These equations are coupled by two sets of common boundary conditions which are determined by noting that both the temperature and the energy flux must be continuous across any transverse plane through the belt. It follows that

$$T|_{x=a^-} = T|_{x=a^+} \qquad (3)$$

$$T|_{x=0} = T|_{x=l} \qquad (4)$$

$$\left.\frac{dT}{dx}\right|_{x=a^-} = \left.\frac{dT}{dx}\right|_{x=a^+} \qquad (5)$$

$$\left.\frac{dT}{dx}\right|_{x=0} = \left.\frac{dT}{dx}\right|_{x=l}. \qquad (6)$$

The governing equations and their boundary conditions are now nondimensionalized by introducing the following nondimensional variables:

$$\theta \equiv [T]\left[\frac{\rho v c_p t}{\sigma\bar{\varepsilon}l}\right]^{-\frac{1}{3}} \equiv \text{nondimensional temperature}$$

and

$$X \equiv \left[\frac{x}{l}\right] \equiv \text{nondimensional length}.$$

Substituting these variables into equations (1)–(6) yields

$$\varepsilon\frac{d^2\theta}{dX^2} - \frac{d\theta}{dX} - \alpha(\theta - \theta_D) = 0\left(0 \leqslant X \leqslant \frac{a^-}{l}\right) \qquad (7)$$

$$\varepsilon\frac{d^2\theta}{dX^2} - \frac{d\theta}{dX} - \theta^4 = 0 \qquad \left(\frac{a^+}{l} \leqslant X \leqslant 1\right) \qquad (8)$$

$$\theta|_{X=a^-/l} = \theta|_{X=a^+/l} \qquad (9)$$

$$\theta|_{X=0} = \theta|_{X=1} \qquad (10)$$

$$\left.\frac{d\theta}{dX}\right|_{X=a^-/l} = \left.\frac{d\theta}{dX}\right|_{X=a^+/l} \qquad (11)$$

$$\left.\frac{d\theta}{dX}\right|_{X=0} = \left.\frac{d\theta}{dX}\right|_{X=1} \quad (12)$$

where

$$\varepsilon \equiv \frac{k}{\rho v c_p l}$$

is the ratio of the conductive energy flux to the radiative energy flux

$$\alpha \equiv \frac{hl}{\rho v c_p t}$$

is the ratio of the energy flux from the drum to the belt over the convected energy flux, θ_D is the nondimensional condenser drum temperature. If the values of the conduction–radiation parameter ε are calculated for typical hardware systems such as those proposed by Weatherston and Smith [1] and Burge [2], it is found that $\varepsilon \ll 1$; that is, the "weak conduction" regime is the one of practical interest. Furthermore, practical considerations also dictate that the product $\varepsilon\alpha$ always approaches zero as ε approaches zero.

Equation (7) is a linear, second order, ordinary differential equation with constant coefficients whose solution is easily shown to be

$$\theta(X) = \theta_D + A e^{m_1(X - a/l)} + B e^{m_2 X} \quad \left(0 \leqslant X \leqslant \frac{a}{l}\right) \quad (13)$$

where

$$m_{1,2} = \frac{1}{2\varepsilon} \pm \frac{1}{2\varepsilon}\sqrt{(1 + 4\varepsilon\alpha)}. \quad (14)$$

Equation (8) may be classified in the same manner except that it is nonlinear and does not possess a solution in terms of known functions. The approximate solution of this equation by singular perturbation theory and the use of the boundary to couple this solution to that given by equation (13) constitute the main problems considered in this study.

2.2 *Outer expansion*

It is now assumed that the temperature distribution may be represented by a straightforward, or outer, asymptotic expansion in integral powers of the perturbation parameter ε, that is,

$$\theta(X;\varepsilon) = \theta_0(X) + \varepsilon\theta_1(X) + \varepsilon^2\theta_2(X) + \ldots (\text{as } \varepsilon \to 0, X \text{ fixed}) \quad (15)$$

Substituting this expansion into equation (8), gathering terms of like powers of ε, and noting that in order for this result to hold for arbitrary values of ε that all of the coefficients must identically vanish, we obtain the following infinite set of "outer" equations:

$$\varepsilon^0: \frac{d\theta_0}{dX} + \theta_0^4 = 0 \quad (16a)$$

$$\varepsilon^1: \frac{d\theta_1}{dX} + 4\theta_0^3\theta_1 = \frac{d^2\theta_0}{dX^2} \quad (16b)$$

$$\varepsilon^2: \frac{d\theta_3}{dX} + 4\theta_0^3\theta_2 = \frac{d^2\theta_1}{dX^2} - 6\theta_0^2\theta_1^2. \quad (16c)$$

Note that the assumption of a straightforward expansion yields a system of *first-order* differential equations whereas the original differential equation (8) was of *second order*. Thus, the order of the governing equation has been reduced from two to one, and both of the original boundary conditions can no longer be satisfied. This loss of the capability of satisfying both boundary conditions marks the appearance of a singular boundary value problem [9] which arises from the fact that the small parameter ε multiplies the highest order derivative in the original differential equation.

A more detailed explanation of this phenomena may be obtained through further examination of equation (8), noting that $a/l \ll 1$, and observing that for reasons of thermal efficiency belt radiators are designed so that the temperature is O(1) over the entire belt. This equation was nondimensionalized such that each term, independent of its coefficient, is of O(1) and thus the relative magnitudes of the terms are determined by the coefficients themselves. Since the coefficient of the conduction term is the small parameter ε, conduction is completely negligible in the lowest order solution and a radiation–convection balance exists over the largest part of the belt not in contact with the condenser drum. Because $a/l \ll 1$, however, an element of the belt in contact with the drum is raised from its lowest temperature near $X = 0$ to its highest temperature near $X = a/l$ over only a small percentage of the total belt length. This causes a relatively high rate of change of the temperature gradient to occur in the vicinity of $X = 1$. Thus, near $X = 1$ the second derivative is not $O(1)$ but becomes just as large as ε is small such that their product is $O(1)$ and the conduction term can no longer be neglected. This is, of course, classical "boundary-layer type" behavior and it requires that the governing equation be rescaled to include the effect of conduction in the vicinity of $X = 1$.

2.3 *Inner expansion*

We define the following "inner" variables as follows:

$$\tilde{X} \equiv \frac{1 - X}{\varepsilon} \quad \text{and} \quad \tilde{\theta}(\tilde{X}) \equiv \theta(X).$$

Substituting these variables into equation (8), we have

$$\frac{d^2\tilde{\theta}}{d\tilde{X}^2} + \frac{d\tilde{\theta}}{d\tilde{X}} - \varepsilon\tilde{\theta}^4 = 0. \tag{17}$$

It is now assumed that $\tilde{\theta}$ may be expanded for small ε in the form:

$$\tilde{\theta}(\tilde{X}\,;\varepsilon) = \tilde{\theta}_0(\tilde{X}) + \varepsilon\tilde{\theta}_1(\tilde{X}) + \varepsilon^2\tilde{\theta}_2(\tilde{X}) + \ldots (\text{as } \varepsilon \to 0, \tilde{X} \text{ fixed}). \tag{18}$$

In a manner similar to that employed in the outer region, we substitute this expansion into equation (8) and obtain the following infinite set of "inner" equations:

$$\varepsilon^0: \frac{d^2\tilde{\theta}_0}{d\tilde{X}^2} + \frac{d\tilde{\theta}_0}{d\tilde{X}} = 0 \tag{19a}$$

$$\varepsilon^1: \frac{d^2\tilde{\theta}_1}{d\tilde{X}^2} + \frac{d\tilde{\theta}_1}{d\tilde{X}} = \tilde{\theta}_0^4 \tag{19b}$$

$$\varepsilon^2: \frac{d^2\tilde{\theta}_2}{d\tilde{X}^2} + \frac{d\tilde{\theta}_2}{d\tilde{X}} = 4\tilde{\theta}_0^3\tilde{\theta}_1. \tag{19c}$$

The boundary conditions for the outer problem are given by equations (9) and (11). The boundary conditions for the inner problem are given by equations (10) and (12). Rewriting these latter equations in inner variables, gives

$$\tilde{\theta}|_{\tilde{X}=0} = \theta|_{X=0} \tag{20}$$

$$\left.\frac{d\tilde{\theta}}{d\tilde{X}}\right|_{\tilde{X}=0} = -\varepsilon\left.\frac{d\theta}{dX}\right|_{X=0}. \tag{21}$$

The right-hand members of equations (9), (11), (20) and (21) may now be evaluated by means of equation (13). The results are

$$\theta|_{X=a^+/l} = \theta_D + A + Be^{m_2a/l} \tag{22a}$$

$$\left.\frac{d\theta}{dX}\right|_{X=a^+/l} = Am_1 + Bm_2e^{m_2a/l} \tag{22b}$$

$$\tilde{\theta}|_{\tilde{X}=0} = \theta_D + Ae^{-m_1a/l} + B \tag{23a}$$

$$\left.\frac{d\tilde{\theta}}{d\tilde{X}}\right|_{\tilde{X}=0} = -\varepsilon[Am_1e^{-m_1a/l} + Bm_2]. \tag{23b}$$

Since A, B, m_1 and m_2 are all functions of ε; however, equations (22) and (23) must be rewritten to explicitly display the dependence of their righthand members on ε. Let

$$A = A_0 + \varepsilon A_1 + \varepsilon^2 A_2 + \ldots \quad (\text{as } \varepsilon \to 0) \tag{24a}$$

$$B = B_0 + \varepsilon B_1 + \varepsilon^2 B_2 + \ldots \quad (\text{as } \varepsilon \to 0). \tag{24b}$$

Expanding the expressions given by equation (14) for m_1 and m_2 for small ε, we obtain

$$m_1 = \frac{1}{\varepsilon} + \alpha - \varepsilon\alpha^2 + 2\varepsilon^2\alpha^3 - 5\varepsilon^3\alpha^4 + \ldots \tag{25a}$$

and

$$m_2 = -\alpha + \varepsilon\alpha^2 - 2\varepsilon^2\alpha^3 + 5\varepsilon^3\alpha^4 + \ldots \tag{25b}$$

It is important to note that, for a given value of the small parameter ε, these expansions place a limiting value on the other parameter in the problem, α, that is, the inequality

$$(4\varepsilon\alpha)^2 < 1$$

must always be satisfied.

Inspection of equation (25a) reveals that the exponential term $e^{-m_1 a/l}$ does not possess a Taylor series expansion about $\varepsilon = 0$ and in fact this term is transcendentally small, that is,

$$e^{-m_1 a/l} \to 0 \qquad (\text{as } \varepsilon \to 0). \qquad (26)$$

Expanding $e^{m_2 a/l}$ for small ε, we have

$$e^{m_2 a/l} = [e^{-\alpha a/l}][1 + \alpha^2(a/l)\varepsilon - 2\alpha^3(a/l)\varepsilon^2 + \tfrac{1}{2}\alpha^4(a/l)^2\varepsilon^2 + \ldots] \qquad (\text{as } \varepsilon \to 0). \qquad (27)$$

Substituting equations (24)–(27) into equations (22) and (23) and grouping the coefficients of like powers of ε and requiring that they vanish, we obtain the following infinite sets of boundary conditions for the outer problems:

$$\varepsilon^0: \theta_0|_{X=a^+/l} = \theta_D + B_0 e^{-\alpha a/l} \qquad (28a)$$

$$\left.\frac{d\theta_0}{dX}\right|_{X=a^+/l} = A_1 - \alpha B_0 e^{-\alpha a/l} \qquad (28b)$$

$$\varepsilon^1: \theta_1|_{X=a^+/l} = A_1 + \alpha^2(a/l)e^{-\alpha a/l}B_0 + e^{-\alpha a/l}B_1 \qquad (28c)$$

$$\left.\frac{d\theta_1}{dX}\right|_{X=a^+/l} = \alpha A_1 + A_2 + \alpha^2 \left(1 - \alpha\frac{a}{l}\right)e^{-\alpha a/l}B_0 - \alpha e^{-\alpha a/l}B_1 \qquad (28d)$$

$$\varepsilon^2: \theta_2|_{X=a^+/l} = A_2 + \left(2\alpha^3\frac{a}{l}\right)\left(\frac{1}{4}\alpha\frac{a}{l} - 1\right) e^{-\alpha a/l}B_0 + \alpha^2\frac{a}{l}e^{-\alpha a/l}B_1 + e^{-\alpha a/l}B_2 \qquad (28e)$$

$$\left.\frac{d\theta_2}{dX}\right|_{X=a^+/l} = -\alpha^2 A_1 + \alpha A_2 + A_3 - \alpha e^{-\alpha a/l}B_2 + [-\tfrac{1}{2}\alpha^5(a/l)^2 + 3\alpha^4(a/l) - 2\alpha^3]e^{-\alpha a/l}B_0 + (\alpha^2)\left(1 - \alpha\frac{a}{l}\right)e^{-\alpha a/l}B_1. \qquad (28f)$$

It should be noted that equation (22b) has given $A_0 = 0$. Similarly, for the inner problems, we obtain

$$\varepsilon^0: \tilde{\theta}_0|_{\tilde{X}=0} = \theta_D + B_0 \qquad (29a)$$

$$\left.\frac{d\tilde{\theta}_0}{dX}\right|_{\tilde{X}=0} = 0 \qquad (29b)$$

$$\varepsilon^1: \tilde{\theta}_1|_{\tilde{X}=0} = B_1 \qquad (29c)$$

$$\left.\frac{d\tilde{\theta}_1}{d\tilde{X}}\right|_{X=0} = \alpha B_0 \qquad (29d)$$

$$\varepsilon^2: \tilde{\theta}_2|_{X=0} = B_2 \qquad (29e)$$

$$\left.\frac{d\tilde{\theta}_2}{d\tilde{X}}\right|_{X=0} = \alpha(B_1 - \alpha B_0). \qquad (29f)$$

2.4 *Uniformly-valid approximation*

It is now possible to understand the basic structure of the overall problem. The n-th order solution requires the determination of five constants: A_n, B_n, the two arbitrary constants in the solution of the inner equation, and one arbitrary constant in the solution of the outer equation. Since A_0 is determined in the derivation of the boundary conditions to be identically zero, however, one of the five constants found in the n-th order solution will always be applicable to the $(n + 1)$-th order solution. Four of the constants are determined by the n-th order boundary conditions while the fifth constant is found by matching [8] the inner and outer asymptotic expansions. Finally, the inner and outer solutions are combined by the additive method [8] to form uniformly valid solutions for the temperature distribution in the segment of the belt not in contact with the condenser drum. The results of carrying out these straightforward operations to second-order are listed below.

Second-order solution

$$\theta(X) = \theta_D + (\varepsilon A_1 + \varepsilon^2 A_2)e^{m_1(X - a/l)} + (B_0 + \varepsilon B_1 + \varepsilon^2 B_2)e^{m_2 X} \qquad \left(0 \leqslant X \leqslant \frac{a^-}{l}\right) \qquad (30a)$$

$$\theta(X) = \left[C_2 + 3\left(X - \frac{a}{l}\right)\right]^{-\frac{1}{3}}$$

$$+ \varepsilon C_3[1 - 4C_1^3(1 - X)]e^{(X - 1/\varepsilon)}$$

$$+ \varepsilon\left[C_2 + 3\left(X - \frac{a}{l}\right)\right]^{-\frac{4}{3}}$$

$$\times \left\{\frac{4}{3}\ln\left[C_2 + 3\left(X - \frac{a}{l}\right)\right] + C_4\right\}$$

$$+ \varepsilon^2 C_6 e^{(X - 1/\varepsilon)} + \varepsilon^2\left[C_2 + 3\left(X - \frac{a}{l}\right)\right]^{-}$$

$$\times \left\{\left[\frac{28}{3} - 4C_4 + 2C_4^2\right]\right.$$

$$+ \left(\frac{16}{3}\right)(C_4 - 1)\ln\left[C_2 + 3\left(X - \frac{a}{l}\right)\right]^{-\frac{1}{3}}$$

$$\left. + \frac{32}{9}\left(\ln\left[C_2 + 3\left(X - \frac{a}{l}\right)\right]\right)^2\right\}$$

$$+ \varepsilon^2 C_7\left[C_2 + 3\left(x - \frac{a}{l}\right)\right]^{-\frac{4}{3}}$$

$$\left(\frac{a^+}{l} \leqslant X \leqslant 1\right). \qquad (30b)$$

The constant B_0 must be found as a root of

$$\left[(\theta_D + B_0)^{-3} + 3\left(\frac{a}{l} - 1\right)\right]^{-\frac{1}{3}} = \theta_D + B_0 e^{-\alpha a/l}. \qquad (31)$$

The remaining constants may then be calculated by means of the following expressions:

$$C_1 = \theta_D + B_0 \qquad (32)$$

$$C_2 = C_1^{-3} + 3(a/l - 1) \qquad (33)$$

$$A_1 = \alpha B_0 e^{-\alpha a/l} - C_2^{-\frac{4}{3}} \qquad (34)$$

$$C_3 = C_1^4 - \alpha B_0 \qquad (35)$$

$$B_1 = [e^{-\alpha a/l} - C_1^{-4} C_2^{-\frac{4}{3}}]^{-1} \times \left\{C_2^{-\frac{4}{3}}\left(\frac{4}{3}\ln C_2 + \alpha B_0 C_1^{-4} - 1 - \frac{4}{3}\ln[C_2 + 3(a/l - 1)]\right) - A_1 - \alpha^2 B_0 \frac{a}{l} e^{-\alpha a/l}\right\} \qquad (36)$$

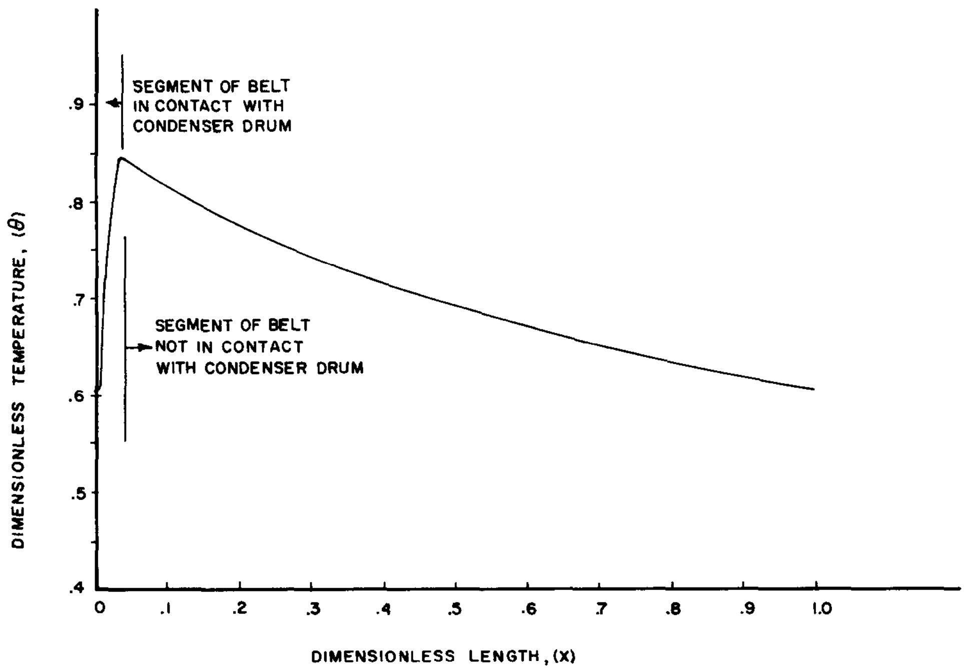

FIG. 2. Second order temperature distribution.

$$C_4 = C_1^{-4}(B_1 + \alpha B_0) - 1 - \frac{4}{3}\ln\left[C_2 + 3(a/l - 1)\right] \quad (37)$$

$$A_2 = \alpha B_1 e^{-\alpha/l} - \alpha A_1 - \alpha^2 B_0 (1 - \alpha a/l) e^{-\alpha a/l} - 4C_2^{-\frac{7}{3}}\left[\frac{4}{3}\ln C_2 + C_4 - 1\right] \quad (38)$$

$$C_5 = \alpha B_0 + B_1 - C_1^4 \quad (39)$$

$$C_6 = 4C_1^3(C_5 - C_3 - C_1^4) + \alpha(\alpha B_0 - B_1) \quad (40)$$

$$\begin{aligned} B_2 = {} & [e^{-\alpha a/l} - C_1^{-4}C_2^{-\frac{4}{3}}]^{-1}\{C_2^{-\frac{4}{3}} \\ & \times [C_2^{-1} - C_1^3]\left[\frac{28}{3} - 4C_4 + 2C_4^2\right] \\ & + \frac{16}{3}C_2^{-\frac{4}{3}}[C_4 - 1](C_2^{-1}\ln C_2 \\ & - C_1^3 \ln[C_2 + 3(1 - a/l)]) \\ & + \frac{32}{9}C_2^{-\frac{4}{3}}[C_2^{-1}(\ln C_2)^2 - C_1^3 \\ & \times (\ln[C_2 + 3(1 - a/l)])^2] - C_1^{-4}C_2^{-\frac{4}{3}}C_6 \\ & - A_2 - 2\alpha^3 B_0 \frac{a}{l}\left(\frac{1}{4}\alpha\frac{a}{l} - 1\right)e^{-\alpha a/l} \\ & - \alpha^2 B_1 (a/l) e^{-\alpha a/l}\} \end{aligned} \quad (41)$$

$$\begin{aligned} C_7 = {} & C_1^{-4}(B_2 - C_6) - C_1^3\left(\frac{28}{3} - 4C_4 + 2C_4^2\right) \\ & - \frac{32}{9}C_1^3\{\ln[C_2 + 3(1 - a/l)]\}^2 \\ & - \frac{16}{3}C_1^3(C_4 - 1)\ln[C_2 + 3(1 - a/l)]. \end{aligned} \quad (42)$$

2.5 *Heat transfer capacity*

These temperature distributions may now be used to calculate the heat transfer capacity of the radiator to the desired order. Since the steady state case is being analyzed, we may write:

$$\dot{q}_{\text{drum to belt}} = \dot{q}_{\text{belt to environment}} = \dot{q}$$

$\equiv$ heat transfer capacity of the system.

For the belt in contact with the condenser drum,

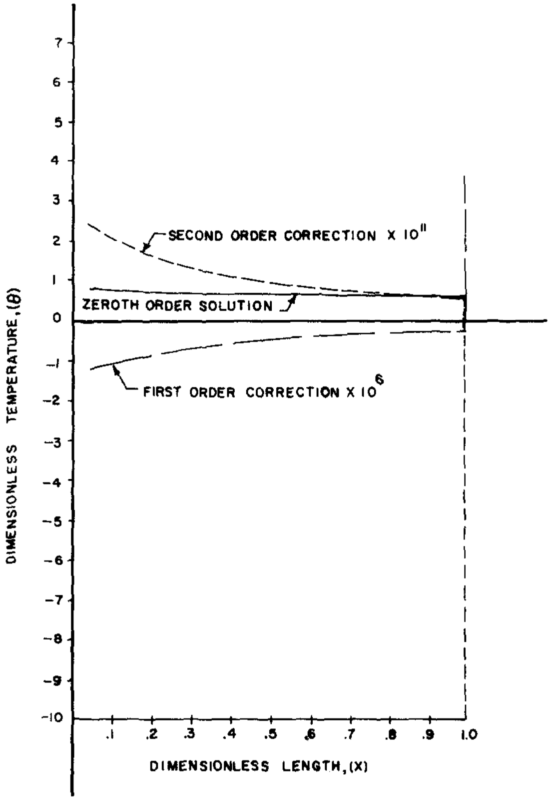

FIG. 3. Comparison of the zeroth-order temperature distribution with its first- and second-order corrections.

we obtain

$$\dot{q} = Wh\int_0^a [T_D - T(x)]\,dX. \quad (43)$$

Nondimensionalizing equation (43), we have

$$\dot{Q} = \int_0^{(a-/l)} [\theta_D - \theta(X)]\,dX. \quad (44)$$

Substituting the temperature distribution given by equation (13) and integrating yields

$$\dot{Q} = -\left(\frac{A}{m_1}\right) + \left(\frac{B}{m_2}\right)(1 - e^{m_2 a/l}). \quad (45)$$

This expression may be expanded to any desired order of ε.

3. RESULTS

In order to exhibit the qualitative trends of the

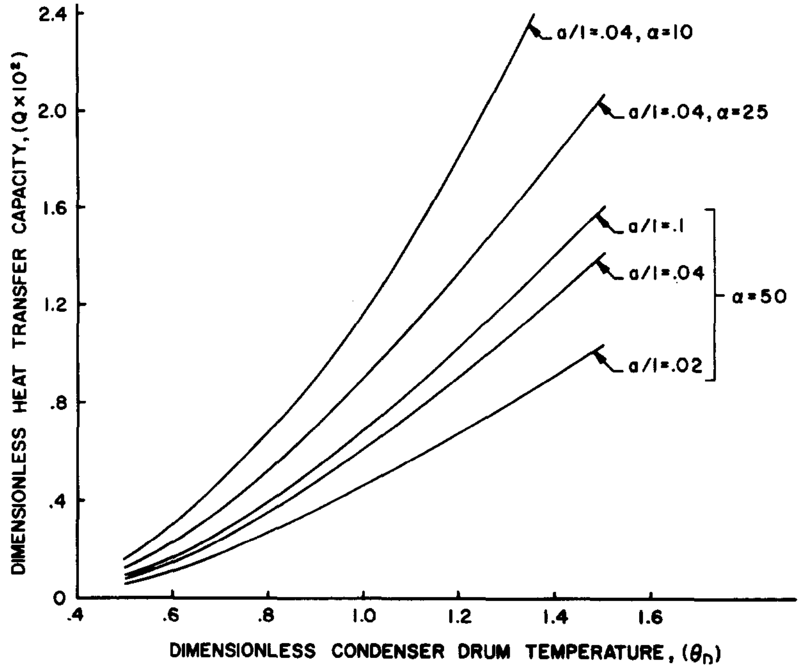

FIG. 4. Heat transfer capacity of a typical belt type radiator.

above solution, the second-order temperature distribution was calculated for a system similar to the one proposed by Burge [3]. This system is defined by the following set of physical variables:

$$\bar{\varepsilon} = 1{\cdot}0 \text{ (black body)}$$
$$c_p = 0{\cdot}1 \text{ (Btu/lb}_m\text{°R)}$$
$$\rho = 500 \text{ (lb}_m\text{/ft}^3\text{)}$$
$$k = 50 \text{ (Btu/hft°R)}$$
$$T_D = 1800 \text{ (°R)}$$
$$h = 720 \text{ (Btu/hft}^2\text{°R)}$$
$$v = 20 \text{ (ft/s)}$$
$$t = 0{\cdot}0002 \text{ (ft)}$$
$$l = 50 \text{ (ft)}$$
$$a = 2 \text{ (ft)}.$$

These variables give a value of (0·277) (10^{-6}) for the small parameter, ε.

The results of these calculations are given in Fig. 2. They show that energy transfer from the condenser drum to the belt by conduction rapidly raises the belt temperature to its maximum value, utilizing only a small percentage of the total belt length to accomplish this task. Radiation to space then takes place over the remainder of the belt and causes the temperature to decay to its minimum value, whereupon the belt is again brought in contact with the drum. Figure 3 shows a comparison of the zeroth-order solution of the problem with its first and second-order correction terms. As can easily be seen, conduction has very little effect on the temperature distribution of the particular system analyzed here.

The effects of variations in the system parameters on the heat transfer capacity of the radiator are shown in Fig. 4. Higher order corrections have been neglected in the parametric study since these corrections were shown to be negligible for systems of practical interest.

4. CONCLUDING REMARKS

The effects of conduction on the temperature distribution in a representative system have been shown to be negligible. This was accomplished in a rational, systematic fashion by formulating the problem within the framework of singular perturbation theory. Thus, the zeroth-order solution, which completely neglects conduction in the belt, should yield suitably accurate solutions for engineering purposes. The higher-order solutions are available, however, to assess the effects of conduction on any system possessing a set of physical variables that differ radically from those of the representative system.

REFERENCES

1. R. C. WEATHERSTON and W. E. SMITH, A method for heat rejection from space powerplants, *ARS Jl* **30**, 268–269 (1960).
2. H. L. BURGE, Revolving belt space radiator, *ARS Jl* **32**, 1243–1248 (1962).
3. R. C. WEATHERSTON and W. E. SMITH, A new type of thermal radiator for space vehicles, *Aeronaut. Engng* **20**, 1 (1961).
4. T. J. MCGEAN, Analysis of a belt type heat transport device, *AIAA J. Spacecraft* **6**, 947–948 (1969).
5. W. LICK, Energy transfer by radiation and conduction, Proc. Heat Trans., Fluid Mech. Inst., No. 14–24 (1963).
6. H. F. MUELLER and N. D. MALMUTH, Temperature distributions in radiating heat shields by the method of singular perturbations, *Int. J. Heat Mass Transfer* **8**, 915–920 (1965).
7. N. D. MALMUTH, M. KASCIC and H. F. MUELLER, Asymptotic and numerical solutions for nonlinear conduction in radiating heat shields, *J. Heat Transfer* **92**, 264–268 (1970).
8. M. VAN DYKE, *Perturbation Methods in Fluid Mechanics*. Academic, New York (1964).
9. J. D. COLE, *Perturbation Methods in Applied Mathematics*. Ginn Blaisdell, Waltham, Massachusetts (1968).

ETUDE THERMIQUE D'UN RADIATEUR DE TYPE COURROIE PAR LA METHODE DES DEVELOPPEMENTS ASYMPTOTIQUES

Résumé—On a obtenu des expressions analytiques pour la distribution de température et la capacité de transfert thermique d'un radiateur de type courroie. On montre que le problème se réduit à deux équations différentielles du second ordre dont les solutions sont couplées par deux systèmes communs de conditions aux limites. L'une de ces équations n'est pas linéaire et ne peut être résolue analytiquement en termes de fonctions connues. On a employé une théorie de perturbation pour obtenir des solutions de cette équation uniformément valables pour des petites valeurs du paramètre conduction rayonnement ε. Afin de montrer le comportement qualitatif de ces expressions, la distribution de température a été calculée jusqu'au second ordre pour un système représentatif.

THERMISCHE ANALYSE EINES STREIFENFÖRMIGEN STRAHLERS NACH DER METHODE DER ÜBEREINSTIMMENDEN ASYMPTOTISCHEN EXPANSIONEN

Zusammenfassung—Analytische Formeln wurden erhalten für die Temperaturverteilung und die Wärmeübertragungskapazität eines streifenförmigen Strahlers.

Das Problem wird auf zwei gewöhnliche Differentialgleichungen zweiter Ordnung reduziert, deren Lösungen durch zwei gewöhnliche Randbedingungen gekoppelt sind. Eine dieser Gleichungen ist nichtlinear and kann nicht mit Hilfe bekannter Funktionen analytisch gelöst werden.

Hier wird die Störungsrechnung angewendet, um allgemeingültige Lösungen dieser Gleichung für kleine Werte des Strahlungs-Leitungs-Parameters ε zu erhalten. Um das qualitative Verhalten dieser Ausdrücke zu zeigen, wird die Temperaturverteilung für ein repräsentatives System bis zur zweiten Ordnung exakt berechnet.

ТЕРМИЧЕСКИЙ АНАЛИЗ ЛЕНТОЧНОГО ИЗЛУЧАТЕЛЯ МЕТОДОМ СРАЩИВАЕМЫХ АСИМПТОТИЧЕСКИХ РАЗЛОЖЕНИЙ

Аннотация—Получены аналитические выражения для распределения температуры и переноса тепла от ленточного излучателя. Показано, что задача сводится к двум обыкновенным дифференциальным уравнениям второго порядка, решения которых объеденены двумя общими системами граничных условий. Одно из этих уравнений является нелинейным и не может быть решено аналитически с помощью известных функций. Для определения равномерно-сходящихся решений этого уравнения при малых значениях лучисто-кондуктивного параметра ε используется сингулярная теория возмущений. Для иллюстрации качественного поведения этих выражений рассчитывается температурное распределение в типичном случае с точностью до 2-го порядка.